Surface Gap Solitons at a Nonlinearity Interface in the Periodic Schrödinger Equation

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Abstract: In the first part of the talk I will present a study of solitary waves localized at the interface of two nonlinear periodic media with different coefficients of the cubic nonlinearity in the one and two-dimensional periodic Schrödinger equation

$$iu_t + \nabla^2 u - V(x)u + \Gamma(x)|u|^2u = 0, \quad x \in \mathbb{R}^d, d = 1, 2$$

$$\Gamma = \Gamma_- \text{ for } x_1 \leq 0, \quad \Gamma = \Gamma_+ \text{ for } x_1 > 0,$$

where $V$ is $2\pi$ periodic in each variable and $\Gamma_+ \neq \Gamma_-$ are real constants. The model is applicable in the field of nonlinear photonic crystals as well as in Bose-Einstein Condensates (BECs). In photonics applications such a structure corresponds to a cubically nonlinear photonic crystal with different values of the nonlinear refractive index on each side of the interface $x_2 = 0$ and in BECs it describes a condensate with different $s$–wave scattering lengths on each side of the interface.

We call solutions $u(x, t) = e^{-i\omega t}\phi(x)$ that are exponentially localized in space surface gap solitons (SGSs) [1,2] as they are inherent to the interface surface and because their propagation constant (or frequency) $\omega$ lies in the band gaps of the corresponding linear operator $L = -\nabla^2 + V(x)$. In our construction SGSs are computed via bifurcation from standard gap solitons (GSs) with $\Gamma \equiv \text{const}$. The SGS continuation curves undergo folds, the location of which is studied analytically in a certain asymptotic regime. Interesting (unexpected) phenomena such as concentration of the SGSs in the less focusing half of the medium are observed.

In the second half of the talk we study the one dimensional system with an interface that is linear as well as nonlinear. The two scenarios of a linear interface we consider are a jump in the value and derivative of $V(x)$ at $x_1 = 0$. We show under which condition each case leads to the occurence of point spectrum of $L$. SGSs then become nonlinear defect modes.

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References: